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# Modeling of Basic Propeller Thrust Test System and Thrust Control Using PID method

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## Abstract

Nowadays, unmanned aerial vehicles take an important place in our life. There are several studies on UAVs. UAVs can be classified according to their propulsion systems like Turbofan UAVs, Turbojet UAVs or Electrical UAVs. Propellers are used to supply the thrust force to Electrical UAVs. It is difficult to obtain available performance data of micro propellers. Also, micro propeller has operated at low Reynold number, so that performance of micro propeller is obtained difficultly. Basic propeller thrust test system is needed to determine the characteristic of propeller. It is known that the characteristic of propeller has great effect on thrust force and torque which are produced by the propeller. Thrust force and torque take important place in UAVs performance. Also, Thrust force can be used to control UAVs in the flight control system. In this paper, PID controller for basic propeller thrust test system was designed and simulated for static thrust condition.

**Key words:** PID, Thrust Force, Propeller, UAV

## 1. Introduction

Recently, multi-rotary wing UAVs such as quadcopter and hexacopter are increasing in the industry. Technological connection between the automotive and aviation industry have been getting stronger since the Wright brothers took flight [1].

Micro scale propellers are generally used in radio controlled (RC) aircraft or multi-rotary wing vehicles. The size and shape of the propellers change according to the type of RC aircraft flown. There are a huge number of RC aircraft which are constructed of foam. These kinds of aircraft are inexpensive and use small motors and propellers with diameters less than 5 in. to provide thrust when they compare with the others. Also, there is increasing interest Micro Air Vehicles (MAVs) for the military applications. Because of that, data on micro propellers is needed [2].

Different control methods have been researched and one of them is Thrust control method [3]. Thrust force of Electrical UAVs can be controlled easily why the output torque can be measured more accurately than the motor current [4]. Therefore, this paper will focus on a thrust control method as a basic study. The control method will be verified by simulation.

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## 2. Modeling of Basic Propeller Thrust Test System

### *2.1. DC Motor*

DC motor is a machine that converts electrical power into mechanical power.

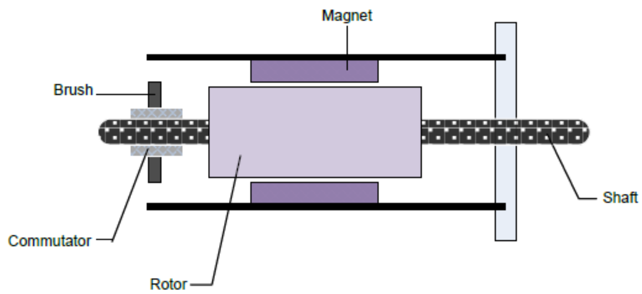



Figure.1 Components of DC motor [5]

A Dc motor has 5 main components as brush, commutator, shaft, magnet, and rotor. Using the Kirchhoff's Voltage Law, the relationship of operation is described [6].

$$V_s = Ri + L\frac{di}{dt} + e \tag{1}$$

where $V_s$ = DC source voltage, i: armature current, e = the back emf.

Also, the mechanical properties of DC motor relative to the torque of system arrangement form Newton's second law. The relationship between inertia load (J), the rate of angular velocity ($\omega_m$) and torque can be described as the following equations [6]:

$$J\frac{d\omega_m}{dt} = \sum T_i \tag{2}$$

$$T_e = k_f \omega_m + J\frac{d\omega_m}{dt} + T_L \tag{3}$$

where,

$T_e$ = the electrical torque, $k_f$ = the friction constant, J = the rotor inertia, $\omega_m$ = the angular velocity, $T_L$ = mechanical load.

Electrical torque and back emf can be described as;

$$e = k_e\omega_m \text{ and } T_e = k_t\omega_m \tag{4}$$

where, $k_e$ = the back emf constant and $k_t$ = the torque constant

## *2.2. Propeller Characterization*

The propulsion unit, shown in Figure 2 consists of a propeller and an electrical motor. The thrust force is produced when a propeller is driven by the electrical motor and also, counter-torque is produced. The value of the thrust force and current depend on the oncoming air velocity and applied voltage. The overall propulsive efficiency are defined as the ratio of the thrust power and the electrical power [7].

$$\eta(V, \upsilon) = \frac{P_{thrust}}{P_{electrical}} = \frac{TV}{i\upsilon} \tag{5}$$

where, $P_{Thrust}$ = the thrust power, $P_{Electrical}$ = the electrical power, T = Thrust force, V = velocity of air, i = current, υ = voltage.

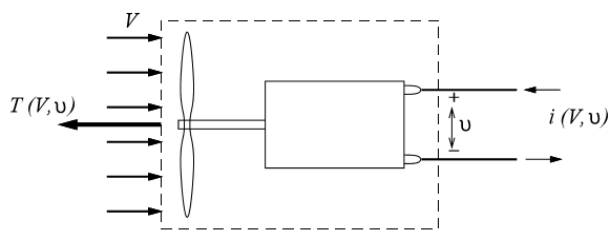



Figure 2. Propulsion unit [7]

Total oncoming flow velocity on the blade airfoil at radius r; depend on air velocity and the rotational speed of the blade [7]. This relationship can be defined as the following equation;

$$W = \sqrt{V^2 + \Omega^2 r^2} \tag{6}$$

The local resultant force has the usual lift and drag components L′ and D′ and also the local resultant force are composed of the thrust and torque components T′ and Q′/r. The overall thrust force and torque values of the propeller are calculating by integrating these T′ and Q′ along the B blades [4].

$$T = B\int_0^R T' dr \tag{7}$$

$$Q = B\int_0^R (Q'/r) r \, dr \tag{8}$$

where T is the thrust force and Q is the counter torque.

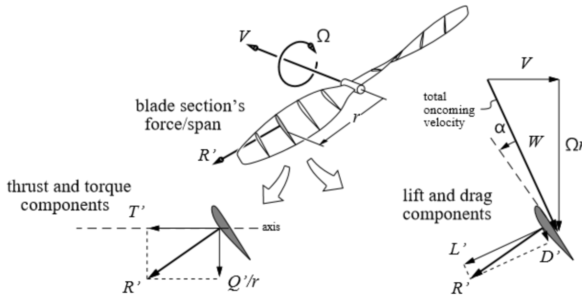



Figure 3. Thrust-torque component and lift-drag component at α angle of attack, on propeller blade [7]

## 3. Designing of Thrust Control of Basic Propeller Thrust Test System

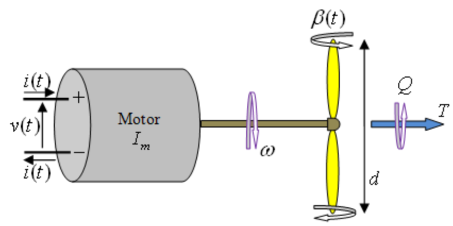



Figure 4. The model of basic propeller thrust test system [8]

### *3.1. Equation of Motion of Propeller*

The propeller cross section is an airfoil. The airfoil can be defined as the geometrical shape, which produces aerodynamic force such as lift and drag force. R is the counter force vector and it depends on the lift and drag forces. The value of the aerodynamic forces are effecting by the angle of attack (α). The angle of advance (φ) is defined as the angle between $V_r$ and the surface of propeller revolution. The angle of attack (α) is represented as [4]:

$$\alpha = \beta - \varphi \tag{9}$$

$$\alpha = \beta - arctan\left[\frac{1}{2\pi r}\frac{V}{n}\right] \tag{10}$$

where β is the angle between the chord of airfoil cross section and the revolution surface, V.

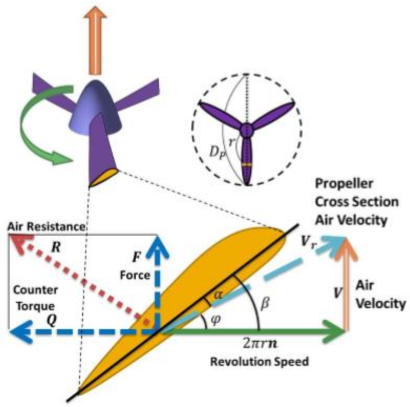


Figure 4. Propeller cross section and effecting forces and torque on propeller [4]

The propeller counter torque (Q) and thrust force (T) can be represented as the following equations [4]:

$$Q = C_Q \rho n^2 D_p^5 \tag{11}$$

$$T = C_T \rho n^2 D_p^4 \tag{12}$$

where ρ is air density.

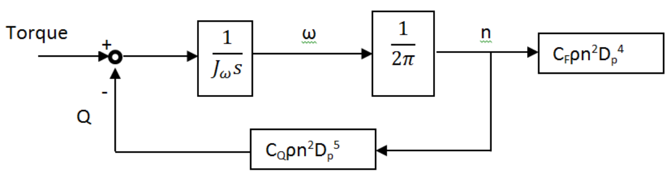


Figure.5 The propeller model

Assuming that the thrust force of the propeller does not change with attitude and the propeller has only rotational movement. The equation of rotational motion of the propeller is defined as [4]:

$$\omega = 2\pi n \tag{13}$$

$$J_p \dot{\omega} = T - Q \tag{14}$$

where $J_p$ is the inertia of propeller, ω is the angular velocity, T is motor torque, Q is the counter torque of propeller [4].

**Table 1.** The Parameters of Propeller

| | |
|---|---|
| **Propeller Diameter ($D_p$)** | 178 mm |
| **Propeller Inertia ($J_p$)** | $1.06\times10^{-5}$ kg.m |
| **Air Density (ρ)** | 1.23 kg/$m^3$ |

### *3.2. Equation of DC Motor*

DC motor can be represented as schematically in Figure 6. The rotor and shaft are assumed as rigid. The physical parameters of DC motor are given in Table 2.

**Table 2.** DC motor parameters

| | |
|---|---|
| **Moment of inertia of the rotor (J)** | 2.14 $gcm^2$ |
| **Damping of the mechanical system (B)** | 0.003 |
| **Electrical Resistance (R)** | 5.88 Ω |
| **Electrical Inductance (L)** | 0.22 mH |
| **Electromotive Force Constant ( Back emf)** | 13.8 mNm/A |
| **Torque constant** | 13.8 mNm/A |

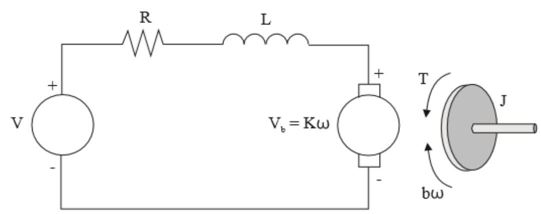



Figure 6. Schematic DC motor [6]

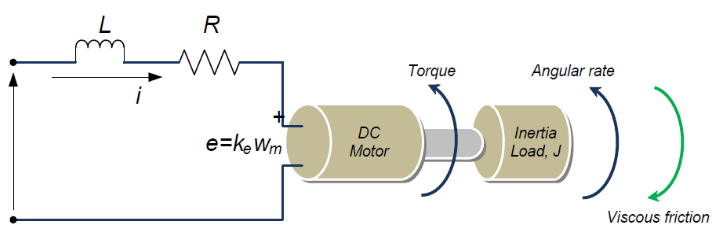



Figure 7. Electromechanical system of DC motor [5]

The motor torque depends on the armature current. The electromotive force also, depends on the angular velocity. The relationships between torque - current, and electromotive force – angular velocity can be describe as the following [5];

$$T = Ki \tag{15}$$

$$V_b = K\omega = \frac{d\theta}{dt} \tag{16}$$

where T = the motor torque, i = the armature current, K = constant factor, $V_b$ = the back electromotive force.

Combining the Newton's second law with Kirchhoff's law:

$$J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} = Ki \tag{17}$$

$$L\frac{di}{dt}+Ri=V-K\frac{d\theta}{dt} \quad (18)$$

Using the Laplace transforms equations (17) and (18) can be re-written as:

$$Js^2\theta(s)+bs\theta(s)=KI(s) \quad (19)$$

$$LsI(s)+RI(s)=V(s)-Ks\theta(s) \quad (20)$$

where s is the Laplace operator.

The equation of DC motor can be express as the following:

$$Js\theta(s)+b\theta(s)=K\frac{V(s)-K\theta(s)}{Ls+R} \quad (21)$$

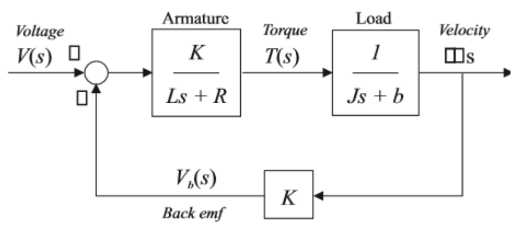



Figure 8. The block diagram of DC motor

The transfer function from the input voltage, V(s), to the angular velocity is express as the following:

$$G_v(s))=\frac{\omega(s)}{V(s)}=\frac{K}{(Ls+R)(Js+b)+K^2} \quad (22)$$

### *3.3. Designing of Thrust Controller*

### *3.3.1. PID Controller*

The PID controller is the most common feedback control method. Today, more than 95% of the control loops have PID method in all areas where control is used; the most of these loops have actually PI control [9].

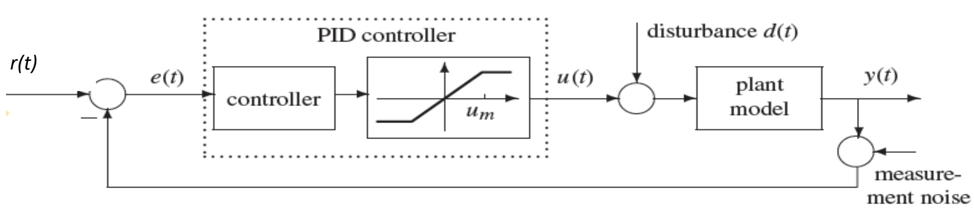



Figure 9. The structure of a PID system

The structure of a PID system is shown in Figure 9 [10]. PID controller can be expressed mathematically as the following equation:

$$u(t) = K_p\, e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt} \tag{23}$$

Where u(t) is the input signal to the plant model, the error signal e(t) is defined as e(t) = r(t) – y(t) is the reference input signal.

Proportional controller ($K_p$) reduces the rise time and the steady-state error. Integral control ($K_i$) eliminates the steady-state errors for a constant or step input but, it makes the transient response slower. Derivative control ($K_d$) increases the stability of the system, reduces the overshoot, and improves the transient response [11].

### *3.3.2. Thrust Controller*

Dynamic model of propeller thrust test system was designed in MATLAB. The Inertia of the dc motor rotor was neglected due to that inertia of propeller is quiet bigger than rotor's when considering inertia of propeller and rotor, also, counter torque was applied to the model as the disturbance effects. Closed Loop System of the propeller thrust test system is shown in Figure 10 and dynamic model of propeller thrust test system is shown in Figure 11.

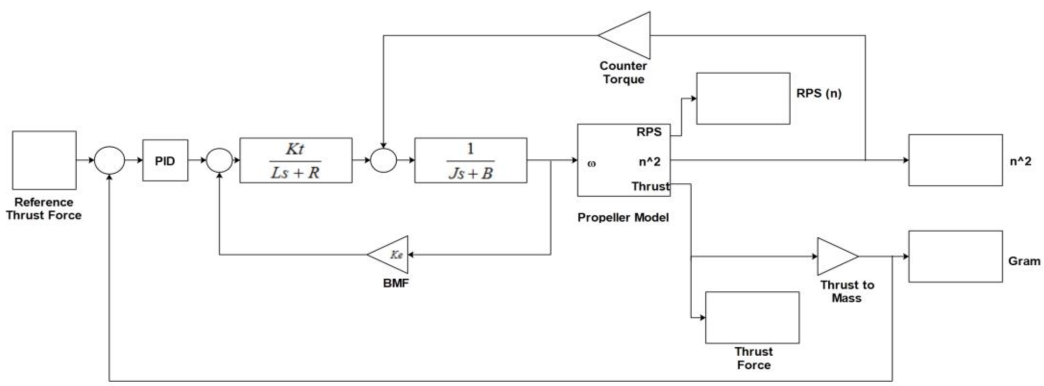



Figure 10. Closed loop system of the propeller thrust test system

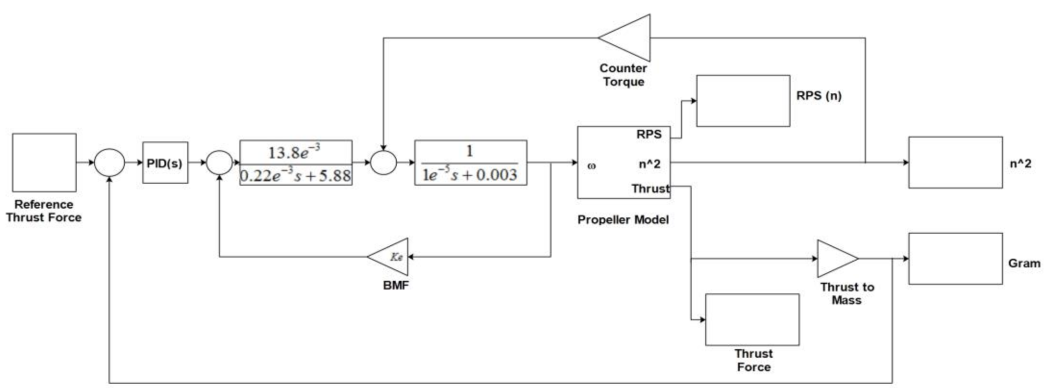
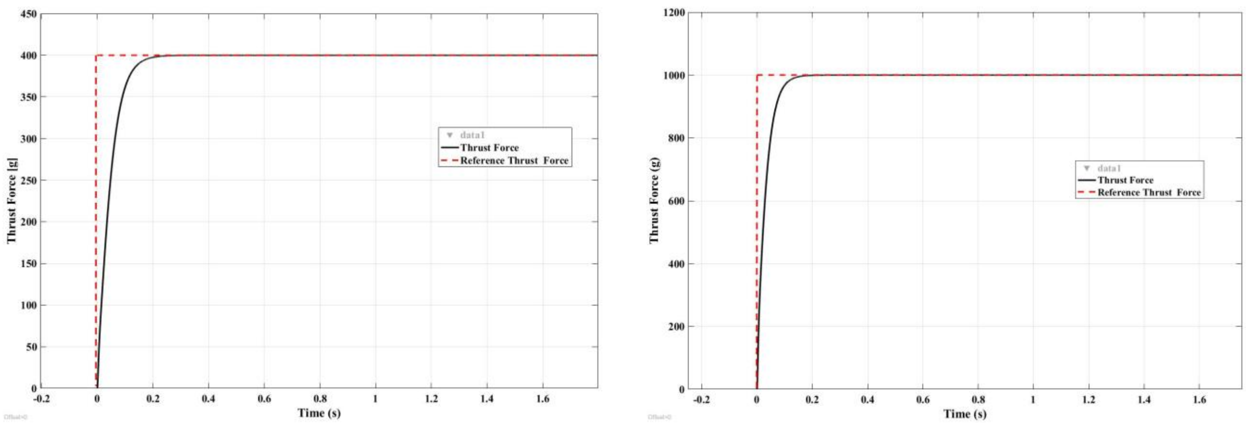



Figure 11. Dynamic model of propeller thrust test system created in MATLAB

## 3. Results

The $k_p$, $k_i$ and $k_d$ gains in PID controller was determined by trial methods. The simulation results for 400 and 1000 g thrust force are shown in Figure 12. The simulations get stable before 0.4 seconds.




(a) (b)

Figure 12. The comparison of reference thrust force and thrust force for 400 g (a) and 1000 g (b).

## Conclusions

In this paper, a thrust control method of propeller for static thrust test system was aimed. First, the single propeller and dc motor was modeled. Next, a thrust control method with a revolution speed control in the inner loop was designed. Finally, simulation was carried out for different thrust forces.

In future studies, experimental setup will be established and also dynamic thrust control system will be designed. More complex model will be designed and robust control methods such as sliding mode will be carried out.

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